ISBN 978-602-6258-17-5

AN APPROACH FOR TIME-DEPENDENT RELIABILITY ANALYSIS OF JACKUP STRUCTURES

Ahmad Idris[1*], Indra Sati Hamonangan Harahap[1] and Montasir Osman Ahmed Ali[1]
[1]Department of Civil and Environmental Engineering, Universiti Teknologi Petronas Bandar Universiti, Seri Iskandar
[*]E-mail : Aidris.civ@buk.edu.ng

ABSTRAK

This paper proposes an approach for evaluation of time dependent reliability of Jackup structures. An approach for signal processing using prolate spheroidal wave functions is combined with stochastic field representation method to represent ocean waves with least number of independent sources of uncertainty. First passage probability for dynamical systems subject to stochastic loading was then used in the formulation of the reliability approach. A simplified Jackup was modelled and used to demonstrate the time dependent reliability approach by propagating the uncertain wave load on the unit. In-house computer codes were developed for the analysis of the stochastic response in time-domain to obtain time dependent failure probabilities. The results were compared with those of a similar model in which the statistical method is used.

Keyword : reliability analysis; offshore structures; time dependent probability.

INTRODUCTION

Jackup is a mobile structure used for oil exploration and production in various offshore locations. Its suitability for use at a specific location is checked using different levels of safety assessment methods. Based on the guideline for site specific assessment (SSA) of Jackup (International Standardization Organization, 2009; SNAME, 2008), when the unit fails to satisfy the criteria of a given assessment level, it is not simply disqualified for use in the location, rather, it is further assessed by increasing the assumptions of the analysis method until a decision can be made. For example, dynamic response can be considered when the unit fails an assessment in which static response was used. One of suchrecommended safety assessment approach is the analysis of the structural response which isusually accomplished by the use of reliability theories whereby a reliability criterion is used to assess the suitability of the units. Such reliability analysis methods are divided in to time dependent and time in-dependent methods. In time independent methods, statistical properties (mean and standard deviation) of the extreme response are used to evaluate the probability that a given response threshold value is not exceeded during the period of excitation. When the unit fails the criteria of time independent approach, appropriate time dependent methods are recommended to be used to in the response analysis.

Jackup reliability analysis using statistical distribution methods (Cassidy, Taylor, Taylor, & Houlsby, 2002; Guedri, Cogan, & Bouhaddi, 2012; Mirzadeh, Kimiaei, & Cassidy, 2016a) is accomplished by selecting appropriate environmental parameters describing a given design state. Structural dynamic analysis is performed on a simplified model (Cassidy, Taylor, & Houlsby, 2001; Jensen & Capul, 2006) or a full model (Mirzadeh, Kimiaei, & Cassidy, 2015, 2016b) in accordance with the SSA guidelines. The statistical distribution function of the extreme response is then used to evaluate the probability that a given response value is not exceeded.

Time dependent reliability analysis approach in Jackup reliability studies is absent in the literature. Presently, the methods used in the response analysis of Jackup units cannot evaluate.

the evolution of the failure probabilities in time domain as recommended in International Standardization Organization (2009), and SNAME (2008). In time dependent reliability analysis, rather than using extreme response values, failure probabilities are evaluated by considering time histories of the structural response due to random stochastic loading. In a fully correlated time dependent problems, failure probabilities are defined over a time interval that consist of a finite number of instants where the variables and failure events between time instants are interdependent (Hu & Du, 2013).

This study aims to propose a methodology for time dependent reliability analysis of Jackup. The method is based on the representation of the sea state using Karhunen-Loeve series (KLS) expansion (Ghanem & Spanos, 1991; Phoon, Huang, & Quek, 2002, 2004). Eigenfunctions of Prolate spheroidal wave functions (PSWF) (Osipov & Rokhlin, 2014) will be used as the orthogonal basis functions of the KLS expansion. Time dependent reliability solution methods will be used in the determination of the failure probabilities of a simplified Jackup model during operation. The results of this study will constitute another level of Jackup SSA in which more complex assumptions are incorporated in to the reliability analysis as recommended in the guidelines.

TIME DEPENDENT RELIABILITY ANALYSIS

When some of the design random variables in reliability analysis are subject to change in properties with due course of time, then the effect of time will have to be featured in the analysis. In such cases, the interest lies in the determination of the probability that the magnitude of the response history of a system will exceed a prescribed threshold level within a given time interval. This is given as (Andrieu-Renaud, Sudret, & Lemaire, 2004):

$$Pf(T)=P\{\exists t\in[0T]:|Y(t)|b\} \quad (1)$$

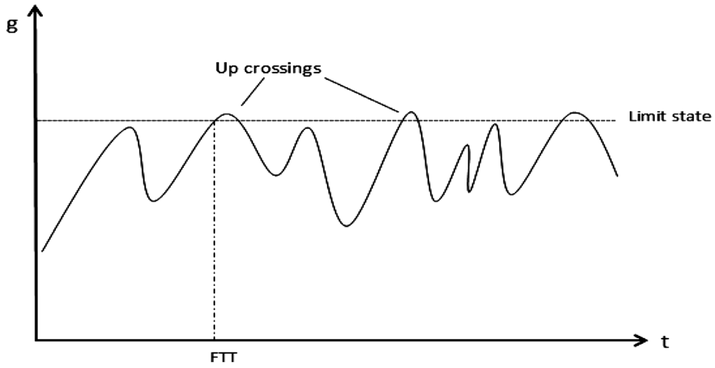
Equation (1) represents the first passage (also known as first excursion) problem with type D barrier whose complexity is related to the interaction of excursion events of the response (He, 2009). During excitation by a stochastic input, the structural response continuously makes transitions from safe to unsafe regions (up crossings) of limit state (Figure 1).

The general approach for the solution of first passage problems is the use of integral equation method based on out crossing theories (Madsen & Krenk, 1984) and Simulation techniques (Dubourg, Sudret, & Deheeger, 2013; Rubinstein & Kroese, 2008).

Simulations techniques involve random simulation of the stochastic input and solving the struc- tural system a number (large) of times (N) to obtain population of structural response (Y(t)). Response variability can then be studied using simple statistical relations as (Sakata, Okuda, & Ikeda, 2015):

ISBN 978-602-6258-17-5

Figure 1. Trajectory of time dependent limit state.



$$E(r_i) = \frac{1}{N}\sum_{I=1}^{N_{MC}} Y_i(j) \quad (2)$$

$$\sigma^2(r_i) = \frac{1}{N}\left[\sum_{I=1}^{N_{MC}} Y_i^{\,2} - N_{MC}E^2(Y_{i)}\right] \quad (3)$$

Other variants of the simulation techniques that aim to reduce the computational cost involved are also employed. For example, methods such as the importance sampling (Papaioannou, Papadimitriou, & Straub, 2016), subset simulation (Schneider, Thöns, & Straub, 2017) and line sam- pling (Lu, Song, Yue, & Wang, 2008) were proposed whereby instead of solving the system a large number of times, the simulation of the random input is controlled in such a way that only repre- sentative random variables are used without significant loss in accuracy. On the other hand, meta- modelling techniques such as the use of Polynomial response surface (Zhao, Fan, & Wang, 2017), gradient-enhanced Kriging (Ulaganathan, Couckuyt, Dhaene, Degroote, & Laermans, 2016) and Artificial Neural Networks (ANN) (Chojaczyk, Teixeira, Neves, Cardoso, & Soares, 2015) are used to develop a model that describe the relationship between the structural input and the resulting output in such a way that when the stochastic input is simulated, the desired population of the response can be obtained without recourse to the structure. For large systems such as Jackup structures, the computational cost required in simulation techniques made its application nearly infeasible.

MODELING OF VARIATIONS IN OCEAN SURFACE ELEVATION

The first stage of uncertainty treatment of systems is the representation of the stochastic fields which is the mathematical description of the uncertain input of the structural system. As it will be seen in the next section, uncertainty analysis of systems subject to stochastic excitation requires the number of dynamic analysis equal to the number of stochastic expansion terms. The Karhunen-Loeve expansion (KLE) method have been shown to be more efficient approach for the representation of a stochastic process with minimal number of independent sources of uncertainty (Idris, Harahap, & Ali, 2017; Phoon et al., 2004). In this study therefore, a method based on KLE that uses orthogonal functions derived from the properties of Prolate Spheroidal Wave Functions (PSWF) will be used in the representation of the wave.

EVALUATION OF PROLATE SPHEROIDAL WAVE FUNCTIONS (PSWFS)

PSWFs plays a significant role in signal processing due to the fact that they have been natural tools for the analysis of band limited functions (Senay, Chaparro, & Durak, 2009). For a sea state in which the spectral energy density of the location is known, the Karhunen-Loeve expansion method with PSWFs can be used to express the surface elevation of the wave to develop wave kinematics and subsequently obtain the wave loading within the duration of the sea state (Sclavounos, 2012).

STRUCTURAL ANALYSIS

A linear structural system is usually represented by an appropriate finite element model with n-degree of freedom. Such systems' structural response due to an input excitation is described by differential equation of the form (Paultre, 2013):

$$M\ddot{x}\,(t, z) + C\dot{x}\,(t, z) + Kx(t, z) = gf\,(t, z) \qquad (4)$$

Here, $\ddot{x}$ and $\dot{x}$ stands for the acceleration and velocity vectors of the response vector xrespectively.
M, C and K are respectively the mass, damping and stiffness matrices which depends on the structural parameters that, for a relatively shorter duration, are assumed to be deterministic and g is the vector associating the random input f(t,z) with the structural degree of freedom.

IMPLEMENTATION OF FRAMEWORK ON IDEALIZED JACKUP UNIT

To demonstrate the application of the framework, a simple Jackup model used in the study inCassidy (1999) was selected and used. The structural and environmental conditions are taken as the same to enable comparison. The model consists of three legs and hull with uniform properties. The legs and hull are considered as beam elements with two legs up-wave and single leg down-wave. The structural material properties of the jack up model can be found in Jensen and Capul (2006), Cassidy et al. (2002) and the effect of marine growth is neglected.

Stationary stochastic ocean condition specifed by Pierson-Markowitz (P-M) spectrum model is assumed. Three design conditions specifed by wave return period and described by signifcant wave height Hs and zero crossing period Tz is considered as given in Table 1.

In uncertainty treatment of stochastic systems, the interest is in the comparison of the response of interest Y(t, z) against the allowable maximum value r* within the period of the excitation. In this study, the response in deck displacement is selected and failure is said to have occurred when the response exceeds the threshold value.

RESULTS AND DISCUSSIONS

The surface elevation of a wave from a sea state simulated for each given return period with KLE expansion using PSWFs by superposition of 8 terms is shown. This is achieved by using the eigenvalues and eigenfunctions of the PSWFs in the expansion.

Table 1. Wave parameters

Return period (Years)	H_S *(m)*	T_Z *(m)*
100	*12.00*	*10.81*
1000	*13.25*	*11.36*
106	*16.45*	*12.66*

The simulated surface elevation was compared with the surface elevation record of a wave obtained under the same environmental conditions and simulated by superposition of 512 terms using New wave theory in the study of Cassidy (1999) as well as using Fourier sine and cosine terms in (Chakrabarti, 1987) with 2000 terms. This is multiple times more than the 8 terms in the PSWFs-KLE method of this study. Consequently, only 8 number of structural analysis runs corresponding to the independent random variables in the KLE representation are required instead of 512 or even 2000 runs if the wavelet superposition method as in the new wave and Fourier method were used. The computer cost required to simulate the wave using the three different methods is shown in (Idris et al., 2017) to be approximately the same.

CONCLUSIONS

As stated in the SSA guidelines (International Standardization Organization, 2009; SNAME, 2008), Jackup suitability for use in a given offshore location is assessed in stages. In the assessment process, one of the methods of the assessment is the use of reliability theories. Time dependent reliability methods are recommended by the guidelines to be used in the assessment. In conclusion, this study has:

- Represented the ocean wave using Karhunen-Loeve expansion with eigenfunctions of prolate spheroidal wave functions in which fewer numbers of independent sources of uncertainty are used.
- Propagated the ocean wave loading on a simplifed Jackup model in the framework of fnite element analysis to obtain dynamic response in deck horizontal displacement.
- Performed time dependent reliability analysis using outcrossing theory and evaluated time dependent probabilities of failure of Jackup unit.The results obtained from the two different timedependent reliability analysis methods have shown that different values of failure probability can be obtained. The comparison made with the results of reliability analysis using extreme response statistics (mean standard deviation) have shown a signifcant difference in values which increases with the increase in the severity of the sea state.

By evaluating the failure probabilities using time dependent reliability analysis methods, this studyhas shown that if a jack up failed the reliability criteria when analysed using statistical methods, then it can be further analysed using the time dependent reliability methods presented in this study. This therefore constitute a further stage of the Jackup SSA using reliability theories as recommended in the guideline. In a similar fashion, failure probability in other criteria such as overturning or bearing capacity can be evaluated using the same approach. By solving the system using any acceptable numerical scheme, the response quantity in any of the criteria can be obtained. The statistical parameters for the response quantity can be evaluated in the same way as those evaluated for the deck displacement in this study, and the reliability analysis can be performed using the same approach.

REFERENCES

Andrieu-Renaud, C., Sudret, B., & Lemaire, M. (2004). The PHI2 method: A way to compute time-variant reliability. Reliability Engineering and System Safety, 84(1), 75–86. https://doi.org/10.1016/j.ress.2003.10.005

Cassidy, M. J. (1999). Non-linear analysis of jack-up structures subjected to random waves (Doctoral dissertation). University of Oxford.

Cassidy, M. J., Taylor, R. E., & Houlsby, G. T. (2001). Analysis of jack-up units using a Constrained New Wave methodology. Applied Ocean Research, 23, 221–234. https://doi.org/10.1016/S0141-1187(01)00005-0

Cassidy, M. J., Taylor, P. H., Taylor, R. E., & Houlsby, G. T. (2002). Evaluation of long-term extreme response statistics of jack-up platforms. Ocean Engineering, 29(13), 1603–1631. https://doi.org/10.1016/S0029-8018(01)00110-X

Chakrabarti, S. (1987). Hydrodynamics of offshore structures.Illinois: Computational Mechanics Publications.

Chojaczyk, A. A., Teixeira, A. P., Neves, L. C., Cardoso, J. B., & Soares, C. G. (2015). Review and application of Artificial Neural Networks models in reliability analysis of steel structures. Structural Safety, 52(PA), 78–89. https://doi.org/10.1016/j.strusafe.2014.09.002

Dubourg, V., Sudret, B., & Deheeger, F. (2013). Metamodel- based importance sampling for structural reliability analysis. Probabilistic Engineering Mechanics, 33, 47–57. https://doi.org/10.1016/j.probengmech.2013.02.002

Ghanem, R. G., & Spanos, P. D. (1991). Stochastic finite elements: A spectral approach. New York, NY: Springer. https://doi.org/10.1007/978-1-4612-3094-6

Guedri, M., Cogan, S., & Bouhaddi, N. (2012). Robustness of structural reliability analyses to epistemic uncertainties. Mechanical Systems and Signal Processing, 28, 458–469. https://doi.org/10.1016/j.ymssp.2011.11.024

He, J. (2009). Numerical calculation for first excursion probabilities of linear systems. Probabilistic Engineering Mechanics, 24(3), 418–425. https://doi.org/10.1016/j.probengmech.2008.12.003

Hu, Z., & Du, X. (2012). Reliability analysis for hydrokinetic turbine blades. Renewable Energy, 48, 251–262.

Hu, Z., & Du, X. (2013). Time-dependent reliability analysis with joint upcrossing rates. Structural and Multidisciplinary Optimization, 48(5), 893–907. https://doi.org/10.1007/s00158-013-0937-2

Idris, A., Harahap, I., & Ali, M. (2017). Efficiency of trigonometric and eigen function methods for simulating ocean wave profile. Indian Journal of Science and Technology, 10(4). doi:10.17485/ijst/2017/ v10i4/110894

International Standardization Organization. (2009). Petroleum and natural gas industries – site-specific assessment of mobile offshore units – part 1: 19905-1.

Jensen, J. J., & Capul, J. (2006). Extreme response predictions for jack-up units in second order stochastic waves by FORM. Probabilistic Engineering Mechanics, 21(4), 330–

337. https://doi.org/10.1016/j.probengmech.2005.11.007 Karoui, A., & Moumni, T. (2008). New efficient methods of

computing the prolate spheroidal wave functions and their corresponding eigenvalues. Applied and Computational Harmonic Analysis, 24(3), 269–289. https://doi.org/10.1016/j.acha.2007.06.004

Lu, Z., Song, S., Yue, Z., & Wang, J. (2008). Reliability sensitivity method by line sampling. Structural Safety, 30(6), 517– 532. https://doi.org/10.1016/j.strusafe.2007.10.001

Lutes, L. D., & Sarkani, S. (2004). Random vibrations: Analysis of structural and mechanical systems. Oxford: Elsevier.

Madsen, P. H., & Krenk, S. (1984). An integral equation method for the first-passage problem in random vibration. Journal of Applied Mechanics, 51(3), 674–679. https://doi.org/10.1115/1.3167691

Mirzadeh, J., Kimiaei, M., & Cassidy, M. J. (2015). A framework to efficiently calculate the probability of failure of dynamically sensitive structures in a random sea. Ocean Engineering, 110, 215–226. https://doi.org/10.1016/j.oceaneng.2015.09.054

Mirzadeh, J., Kimiaei, M., & Cassidy, M. J. (2016a). Effects of irregular nonlinear ocean waves on the dynamic performance of an example jack-up structure during an extreme event. Marine Structures, 49, 148–162. https://doi.org/10.1016/j.marstruc.2016.05.007

Mirzadeh, J., Kimiaei, M., & Cassidy, M. J. (2016b). Performance of an example jack-up platform under directional random ocean waves. Applied Ocean Research, 54, 87–100. https://doi.org/10.1016/j.apor.2015.10.002

Moore, I. C., & Cada, M. (2004). Prolate spheroidal wave functions, an introduction to the Slepian series and its properties. Applied and Computational Harmonic Analysis, 16(3), 208–230. https://doi.org/10.1016/j. acha.2004.03.004

Osipov, A., & Rokhlin, V. (2014). On the evaluation of prolate spheroidal wave functions and associated quadrature rules. Amsterdam: Elsevier Inc.

Papaioannou, I., Papadimitriou, C., & Straub, D. (2016). Sequential importance sampling for structural reliability analysis. Structural Safety, 62, 66–75. https://doi.org/10.1016/j.strusafe.2016.06.002

Paultre, P. (2013). Dynamics of structures. London: John Wiley & Sons. https://doi.org/10.1002/9781118599792

Phoon, K. K., Huang, S. P., & Quek, S. T. (2002). Implementation of Karhunen-Loeve expansion for simulation using a wavelet-Galerkin scheme. Probabilistic Engineering Mechanics, 17(3), 293–303. https://doi.org/10.1016/ S0266-8920(02)00013-9

Phoon, K. K., Huang, H. W., & Quek, S. T. (2004). Comparison between Karhunen-Loeve and wavelet expansions for simulation of Gaussian processes. Computers and Structures, 82, 985–991. https://doi.org/10.1016/j. compstruc.2004.03.008

Rice, S. (1944). Mathematical analysis of random noise. Bell System Technical Journal, 23(3), 282–332. https://doi.org/10.1002/bltj.1944.23.issue-3

Rokhlin, V., & Xiao, H. (2007). Approximate formulae for certain prolate spheroidal wave functions valid for large values of both order and band-limit. Applied and Computational Harmonic Analysis, 22(1), 105–123. https://doi.org/10.1016/j.acha.2006.05.004

Rubinstein, R. Y., & Kroese, D. P. (2008). Simulation and the Monte Carlo method. Wiley, 377.

Sakata, S., Okuda, K., & Ikeda, K. (2015). Stochastic analysis of laminated composite plate considering stochastic homogenization problem. Frontiers of Structural and Civil Engineering, 9(2), 141–153. https://doi.org/10.1007/ s11709-014-0286-2

Schneider, R., Thöns, S., & Straub, D. (2017). Reliability analysis and updating of deteriorating systems with subset simulation. Structural Safety, 64, 20–36. https://doi.org/10.1016/j.strusafe.2016.09.002

Schrupp, K., & Rackwitz, R. (1988). Outcrossing rates of marked poisson cluster processes in structural reliability. Applied Mathematical Modelling, 12(5), 482–490. https://doi.org/10.1016/0307-904X(88)90085-6

Sclavounos, P. D. (2012). Karhunen-Loeve representation of stochastic ocean waves. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 468(2145), 2574–2594. https://doi.org/10.1098/ rspa.2012.0063

Senay, S., Chaparro, L. F., & Durak, L. (2009). Reconstruction of nonuniformly sampled time-limited signals using prolate spheroidal wave functions. Signal Processing, 89(12), 2585–2595. https://doi.org/10.1016/j.sigpro.2009.04.020

SNAME. (2008). Guidelines for site specific assessment of mobile jack-up units. T&R Bullitin 5-5A, 366.

Sudret, B. (2008). Analytical derivation of the outcrossing rate in time-variant reliability problems. Structure and Infrastructure Engineering, 4(5), 353–362. https://doi.org/10.1080/15732470701270058

Ulaganathan, S., Couckuyt, I., Dhaene, T., Degroote, J., & Laermans, E. (2016). Performance study of gradient- enhanced Kriging. Engineering with Computers, 32(1), 15–34. https://doi.org/10.1007/s00366-015-0397-y

Valdebenito, M. A., Jensen, H. A., & Labarca, A. A. (2014). Estimation of first excursion probabilities for uncertain stochastic linear systems subject to Gaussian load.

Computers and Structures, 138, 36–48. https://doi.org/10.1016/j.compstruc.2014.02.010

VanMarcke, E. H. (1975). On the distribution of the first-passage time for normal stationary random

processes. Journal of Applied Mechanics, 42(1), 215. https://doi.org/10.1115/1.3423521

Xiao, H., Rokhlin, V., & Yarvin, N. (2001). Prolate spheroidal wavefunctions, quadrature and interpolation. Inverse Problems, 17(4), 805–838. https://doi.org/10.1088/0266-5611/17/4/315

ISBN 978-602-6258-17-5

Zhao, W., Fan, F., & Wang, W. (2017). Non-linear partial least squares response surface method for structural reliability analysis. Reliability Engineering & System Safety, 161(January), 69–77. https://doi.org/10.1016/j. ress.2017.01.004