INTRODUCTION

Geometry is the discovery of new properties based on facts that can be accepted and recognized by using a deductive reasoning process in a mathematical system (Arifin, 2021). Geometry proof is a series of logical arguments to find answers to why and how to get the statement. Based on this, theorem proving means proving the truth of a mathematical sentence. Mathematics and mathematical proof are two interrelated concepts because in mathematics there needs to be a truth that cannot be represented by only determining true or false (Hermanto et al., 2016). Geometry proof ability is the ability to understand mathematical statements or symbols and compile proof of the truth of a statement mathematically based on definitions, principles, and theorems. As has also been explained that proof in mathematics, especially geometry, is important. It can be said to be important because in mathematics, proof is something that needs to be considered. Learning geometry without being accompanied by proof does not reflect the theory and practice of mathematics.

Geometry course is also a compulsory course course taught by Mathematics Education students at UIN Sunan Gunung Djati Bandung in the odd semester of the first year. Therefore, students must have the ability of mathematical proof, especially in geometry courses in order to construct the principles of geometry with logical arguments. The purpose of students having the ability to provide logical arguments, learning to reason is that students are able to use proof ability is the ability to understand mathematical statements or symbols and compile proof of the truth of a statement mathematically based on definitions, principles, and theorems.

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reasoning in applying patterns and properties, can manipulate, construct evidence and explain mathematical ideas (Jihad, 2018). The error analysis carried out in this study is expected to be used as a basis by lecturers or teachers in providing appropriate assistance to students in avoiding and minimizing errors that are commonly made and can be used as a reflection material to determine what things should be focused on during learning. The results of this study should be used as evaluation material for related parties so that the learning outcomes of geometry courses are well implemented.

In classroom learning, especially when completing mathematical proofs of Geometry courses, students often make mistakes. Several errors were obtained including the consistency of answers and systematic errors that occurred due to the level of mastery of the material that was still lacking in students (Iskandar & Andriyani, 2016). Based on previous research where students showed a negative response when completing the proof of geometry theorems, including most students only rewrite the problems given and not done by students, some students do the problem but when having difficulty then change the next problem number and students do but the proof is done in circles so it does not come to a conclusion (Masfingatin et al., 2018). Errors in solving problems occur one of them because of the low level of mastery of the material, but there are errors that are incidental, namely not reading the will of the problem carefully (Fitriyah, 2016). It can be concluded from the study, the problem found is that students give a poor response in dealing with geometry proof problems. The purpose of error analysis is to repeat the results of student work to identify the form and model of errors in detail and support in choosing priorities in learning.

The function of error identification research according to Brown and Skow in Sulistyorini is to help lecturers to: (1) identifying tasks completed by students to find out which steps students make mistakes, (2) knowing the characteristics of errors made by students, (3) determining the causes of errors that occur only momentarily or basic misconceptions, and (4) accuracy in determining efficient and effective learning approaches in order to make solutions to student misunderstandings (Sulistyorini, 2017). There are 5 categorizations in solving a problem in the problem, namely reading errors, misinterpreting problems, transforming errors, processing ability errors, and errors in preparing the final answer (Csáky et al., 2015). Therefore, in this study, researchers used Newman’s Theory as a reference in identifying student errors in proof. Newman’s Analysis Theory is a suitable method used to analyze answer errors in description problems. When students solve geometry proof problems, where the proof problem is presented in the form of descriptions and the solution must be structured. Error analysis is an important step in improving teaching methods and the student learning experience. By identifying errors, you can take corrective action and develop more effective strategies. The contribution to improving teaching methods is that the results of error analysis can be used as discussion material with colleagues, as a test material for implementing teaching methods that suit the needs of students. In improving students' learning experience, the existence of error analysis is one way to understand the conditions of students who may have difficulties in learning.

Newman’s theory states that there are five specific efforts as a crucial thing to make it easier to get the location of errors and the causes of errors made (Kania & Arifin, 2019). The following error categories in accordance with Newman’s Theory used in this study are: (1) reading errors; (2) comprehension errors; (3) transformation errors; (4) process skills errors; and (5) encoding errors. Based on the exposure of these problems, the relationship between the importance of error analysis is done to minimize errors occur repeatedly with five indicators of Newman Theory error analysis in geometry proof problems is important. Where if not done error analysis is likely to occur the same error repeatedly. As geometry is a course that is the basis and prerequisites for advanced geometry courses. And reinforced by the suggestions of previous researchers who suggested the need for error detection analysis to deepen student difficulties in solving problems using Newman's Theory (Suwanti & Fayeldi, 2018). From the description
of these problems, the researcher aims to analyze the characteristics of errors made by students in solving geometry proof problems based on Newman's Theory.

**METHOD**

This study uses descriptive qualitative methods using phenomenography strategies, because researchers intend to examine the characteristics to minimize student errors in doing proof problems in geometry courses. Phenomenography is a research strategy adapted to map different qualitative ways in which people experience, conceptualize and understand various aspects and phenomena that surround them (Arifin, 2020). The existence of this research is because researchers will observe and analyze existing phenomena. In this study, the identification of phenomena that will be carried out regarding student errors in solving geometry proof problems. The phenomenon will be reviewed based on the type of error analysis of Newman Theory. Research on the phenomenon of errors in solving geometry proof problems is important to be studied because the proof in geometry needs to be tested properly by the arguments put forward logically.

The location of this research was conducted at Mathematics Education, Faculty of Tarbiyah and Teacher Training Sunan Gunung Djati State Islamic University Bandung. The data source is selected according to the research needs, in this study the data source is a first semester student of Mathematics Education Study Program of UIN Sunan Gunung Djati Bandung Academic Year 2019/2020. From the student data source, the researcher will determine the research subject using purposive sampling technique by considering student errors in solving the midterm exam geometry proof based on Newman Theory. Data collection techniques use documentation and interviews. Interview questions were made according to the indicators of error analysis based on Newman's Theory. The documentation used is student answer sheets for proof questions on Midterm Exams, quizzes, and Final Exams. Researchers chose documentation and interviews in order to be able to review and control respondents more intensely with the questions given to respondents and will also better support the documentation of the test results collected.

The subjects of this study were 15 students who were categorized as high class 5 subjects, medium class 5 subjects, and low class 5 subjects. Selection of research subjects with the consideration that students from the high, medium, low categories made five errors according to Newman's Theory error indicators when solving geometry proof problems. Grouping students in high, medium and low ability categories. All subjects of these research made errors in accordance with the type of Newman's Error Theory, following Table 1 which shows a list of research subject codes.

**Table 1. List of Research Subject Codes**

<table>
<thead>
<tr>
<th>No</th>
<th>Subject Code</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T1, T2, T3, T4, T5</td>
<td>High grade research subjects</td>
</tr>
<tr>
<td>2</td>
<td>S1, S2, S3, S4, S5</td>
<td>Medium grade research subjects</td>
</tr>
<tr>
<td>3</td>
<td>R1, R2, R3, R4, R5</td>
<td>Low grade research subjects</td>
</tr>
</tbody>
</table>

Table 2 is the code used in the figure to show the type of Newman's Error Theory.

**Table 2. Code List of Newman Theory Error Types**

<table>
<thead>
<tr>
<th>No</th>
<th>Newman Theory Error Types</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reading errors in the representation of mathematical symbols</td>
<td>RE</td>
</tr>
<tr>
<td>2</td>
<td>Comprehension errors</td>
<td>CE</td>
</tr>
<tr>
<td>3</td>
<td>Transformation errors</td>
<td>TE</td>
</tr>
<tr>
<td>4</td>
<td>Process skills errors</td>
<td>PSE</td>
</tr>
<tr>
<td>5</td>
<td>Encoding errors</td>
<td>ECE</td>
</tr>
</tbody>
</table>

The qualitative data analysis used is Miles and Huberman data analysis which is divided into three, namely: 1) Data reduction, researchers prepare and correct proof problems. Then, select
research subjects according to the criteria of Newman Theory’s error types in each high, medium, and low class category. 2) Presentation of data, researchers present the results of student work selected as research subjects and interview results. 3) Drawing conclusions, researchers compare the analysis of answer sheets of test results and interviews of students who are research subjects using data triangulation techniques.

RESULTS AND DISCUSSION

Results

The stage in this research using the phenomenographic method are the stage of planning, collecting, analyzing, and interpreting data. In the planning stage of this research, what researchers did was to prepare mid-term and final exams questions that have been made by geometry course lecturers. Similarly, quiz questions adapted from the book Geometry for Elementary School. Based on the results of the mid-term test conducted by checking the answers to the proof questions that have been done, fifteen research subjects were obtained consisting of five high category research subjects, five medium category research subjects, and five low category research subjects.

At the data collection stage, the data collected comes from the answer sheet of test results done by research subjects which are then interviews to obtain data accuracy. The answer sheet for geometry test results were analyzed only the items of proof. The data in this study are available in the form of descriptions and tables from the results of test analysis and interviews. The results of the analysis of answer documentation and interviews will be described based on errors in solving geometry proof questions according to Newman’s Theory. The following in Figure 1 is one of the explanations of data analysis of test and interview results in the form of pictures and tables from research subjects.

Figure 1. T1’s Answer to Final Exam Question Number 5

R  :  “Is the symbol writing you used in solving the problem appropriate?”
T1  :  “It doesn’t look like it”
R  :  “Check again”
T1  :  “Something is wrong, this is wrong there should be a u mark”
R  :  “Why not write it down?”
T1  :  “When you’re taking an exam, you just write down what you remember.”
R  :  “How do you plan to proof the problem??”
T1  :  “Yesterday what was taught was like what is known first, then from that, how to elaborate it, find what is the same”
R  :  “How did you solve the problem? Explain each step and why.”
T1: “For the statement, it doesn’t seem to be in order according to my memory. For the reason for the statement I am still confused between postulates or theorems, I don’t know”

R: “Does your answer match the request in the question”

T1: “Looks like it’s been done”

R: “Check question number 5, what is the final proof?”

T1: “$Z$ midpoint $WX$ the reason is the side-angle-side postulate”

R: “Show me the construction”

T1: “$WZ \cong ZX$ and $Z$ is the midpoint, so $WZ$ is a side, $Z$ is an angle dan $ZX$ is a side”

R: “For the quiz question, what is the reason for number 3 and number 4?”

T1: “I’m actually confused between definitions, postulates and theorems”

The results of the analysis in Figure 1 and interviews conducted by researchers on research subject T1 obtained that the errors on T1’s answer sheet included errors in writing the size of the symbols used in statement number 2. Subject T1 acknowledged and knew the location of his mistake. Then another error obtained was an error in determining the proof plan in accordance with the procedure, this error is like in the sequence of each proof statement and does not bring up the main concept of the problem so that this error is included in the error of transforming the proof (transformation errors). Likewise, process skills errors where the research subject T1 was less precise in determining the reason for the statement he had chosen. This resulted in reasoning errors in the final writing of the proof answer (encoding errors). Determination of the reason at the end of this proof statement, T1 did not understand the placement of the reason because the construction shown during the interview was not in accordance with the rules of geometry should be.

<table>
<thead>
<tr>
<th>No</th>
<th>Theory Error Type Newman</th>
<th>High Grade Category</th>
<th>Medium Grade Category</th>
<th>Low Grade Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Reading errors in the representation of mathematical symbols</td>
<td>Students can identify the proof correctly and incompletely write geometry symbols according to the rules.</td>
<td>Students can identify the proof correctly and incompletely write down the geometry symbols according to the rules.</td>
<td>Students are not able to identify correctly and do not write geometry symbols according to the rules.</td>
</tr>
<tr>
<td>2.</td>
<td>Comprehension errors</td>
<td>Students can determine and write what is known, what will be proven and the construction of the drawing.</td>
<td>Students can determine what is known and what will be proven, but do not write hypotheses and construction drawings.</td>
<td>Students can determine what is known and what will be proven, but do not write hypotheses and construction drawings.</td>
</tr>
<tr>
<td>3.</td>
<td>Transformation errors</td>
<td>Students can determine the plan systematic proof.</td>
<td>Students are incomplete in determine the proof plan.</td>
<td>Students are unable to determine a systematic proof plan.</td>
</tr>
<tr>
<td>4.</td>
<td>Process skills errors</td>
<td>Students are not careful in implementing the plan evidence chosen.</td>
<td>Students are not careful in implementing the plan evidence chosen.</td>
<td>Students are not careful in implementing the plan evidence chosen.</td>
</tr>
<tr>
<td>5.</td>
<td>Encoding errors</td>
<td>Students can determine the correct reasoning for the final statement to be proved.</td>
<td>Students are unable to determine the correct reasoning for the final statement to be proved.</td>
<td>Students are unable to determine the correct reasoning for the final statement to be proved.</td>
</tr>
</tbody>
</table>
The interpretation stage contained in Table 3 is the source triangulation data. This triangulation of data sources involves collecting data from a variety of different sources. Researchers collected data from documentation of geometry proof tests and student interviews. By using these various data sources, the researcher can ensure that his findings are supported by evidence from various points of view. This source triangulation is obtained from the results of comparing test results done by high, medium and low grade category students according to Newman Theory error types.

Based on the overall description of the data that has been presented, all research subjects in the high, medium and low-grade categories made mistakes. The majority of research subjects made mistakes in terms of the type of error Newman Theory made, namely errors in proof process skills error. This is most subjects mentioned that they were still confused about determining the right reason for each statement. The minority of research subjects made mistakes in understanding the problem. Overall, the research subjects correctly wrote the initial hypothesis of the problem which include the information found, what would be proven and the construction of the drawing. In the Midterm Exam Questions there are 5 questions and proof questions are found in numbers 2, 3 and 5. Meanwhile, in the Final Exam Questions there are 5 questions and proof questions are found in numbers 1 and 3. Table 4 summarize the types of errors made by research subjects according to Newman's Theory.

**Table 4. Number of Subjects Who Made Errors on Each Item of Geometry Proof Problem**

<table>
<thead>
<tr>
<th>No</th>
<th>Newman Theory Error Types</th>
<th>Number of Subjects Who Made Errors on Each Item of Geometry Proof Problem</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mid No2 Mid No3 Mid No5 Quiz Final No1 Final No3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Reading Errors (RE)</td>
<td>11  10  7  5  2  8  11</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>Comprehension Errors (CE)</td>
<td>1   0   3  1  5  13  5</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>Transformation Errors (TE)</td>
<td>11  12  12 14 9  15  13</td>
<td>73</td>
</tr>
<tr>
<td>4</td>
<td>Process Skills Errors (PSE)</td>
<td>13  14  11 14 13 14 79</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Encoding Errors (ECE)</td>
<td>14  15  13 14 2  64</td>
<td></td>
</tr>
</tbody>
</table>

The results of Table 4 obtained are the percentage for error in reading the mathematical symbol representation (RE) of 15.81%, the percentage of error in comprehension error (CE) of 4.78%. Problem transformation error (TE) with a percentage of 26.84%, percentage of proof process skill error (PSE) of 29.04% and encoding or answer writing error (ECE) of 23.53%. It can be seen that the largest percentage of errors is PSE (Process Skills Errors), one of the process skills errors in determining the right reason for each statement chosen. While the least percentage is CE (Comprehension Errors), errors in understanding the problem in which the majority of research subjects have been able to understand the proof problem well. The average research subject has recorded the known information and what will be prove in the geometry proof problem.

**Discussion**

Based on the results of the identification of test sheets and interviews on research subjects, it can be recognized various errors made by students in working on geometry course proof questions. The results of the data analysis obtained data on the characteristics of errors made by students in accordance with Newman Theory errors. The characteristics of student errors are as follows:

1. Reading errors in the representation of mathematical symbols

Errors in reading mathematical symbol presentations are not only reading problem, but the ability to identify problems or recognize symbolic term in problem. Based on research conducted by Singh that reading errors include student failing to recognize symbols or words in the problem (Singh et al., 2010). The characteristics of the errors found in the research subjects are student do not write geometry symbols according to rules, students
have not recognized the existing geometry terms, students can't identify the proof problem and students write information in the form of statements that are not needed in solving the proof problem. The cause of errors in reading mathematical symbol representations is because students are less careful in writing symbols.

2. Comprehension errors of proof problem
   Problem understanding errors are errors that students make after reading symbol representations but cannot know what problems to solve. This is with Duval’s research which resulted in many students having difficulty in understanding mathematical problems (Duval, 2006). The data obtained from the results of the analysis of the documentation of the answers of the research subjects and the interviews conducted are that students do not know and write completely what is known, what will be proven and the construction of images on the problem. The cause of the comprehension error problem was because during the exam students were in a hurry to solve the problem.

3. Transformation errors of proof problem
   Problem transformation errors, especially in proof, are errors made by students in choosing and determining the right proof plan. Based on previous research, it strengthens that the characteristics of transformation errors are that students are wrong in determining problem-solving strategies (Ekayanti, 2017). The characteristics of this type of error made by students include errors in planning proof, failing to determine and select the right proof procedure and being incomplete in applying the chosen plan. The cause of the error in transforming the proof problem was because they did not master the material and did not understand the concept of a systematic proof plan. When constructing proofs, students need a sequence of implications based on the information already in the problem and write reasons to explain the truth of the statement. Actually, it only takes a few steps of logical proof, but many students make mistakes in these stages due to lack of accuracy in constructing geometry proofs (Arnal-Bailer & Manero, 2023).

4. Process skills errors of proof problem
   Process skill errors are students who make mistakes in the process of implementing the chosen proof concept. Students are able to solve proof problems, but are less complete in expressing statements and less precise in determining the reasons for each statement. The characteristics of errors found from the test data and interviews of research subjects are errors in applying the chosen proof procedure, errors in determining the reasons for each statement, errors in clarifying the truth of statements and reasons, and not being careful in determining each step of the proof. The characteristics are reinforced by Utami's research which found that students have difficulty in writing down their thoughts to determine the reasons and procedures for proof (Utami, 2016). The cause of the proof process skills error was because they could not determine the reason for each statement used.

5. Encoding errors
   Errors in writing the final answer are errors in the problem solving process that causes students not to determine the right reason at the end of the proof. Previous research also mentioned that students did not conclude the final answer correctly in solving geometry problems (Riastuti et al., 2017). The characteristics of this error based on test and interview data are not careful in determining the final reason for proof, not checking back on the answer sheet and not checking back what will be proven, so it is wrong to determine the final result. The cause of encoding errors is because they are not sure about their answers and do not check their work again.

From the findings obtained, it can be said that many errors made by students are process skills errors. Characteristics of process skill errors one of them is in determining the right reason for each statement chosen. This is different from previous researchers by Pamungkas and Wicaksono who produced types of errors in solving plane geometry problems with the largest percentage being in encoding errors and the smallest percentage in reading errors (Pamungkas & Wicaksono, 2019). However, there are also those whose research results are in accordance,
namely the research conduct out by Alhassora. His research found that The results of his research showed that most students were able to do the first and second stages of Newman's Theory, namely reading problems. However, they face difficulties in performing stage two to five of Newman's Theory, namely errors in understanding, transforming problems and solving process skills (Alhassora et al., 2017).

Based on previous research, it is stated that the cause of process skill errors occurs because they have not been able to use their definitions and theorems to develop logical relationships (Masfingatin et al., 2018). This is in accordance with one of the causes found in this study, namely not being careful in writing complete reasons. The solution that can be used to minimize the proof process skill errors made by the research subjects is that students are ensured to understand the intent of the objectives requested in the problem. Likewise, it is necessary to ensure that students understand the basic concepts of geometry so that students can determine the reasons for the statements they choose, and students also need to set the right study time. The solution is reinforced by Nurjanati’s research which states that errors can be minimized if students improve their learning patterns regularly (Nurjanatin et al., 2017).

As for the error analysis research in this course, it can be used as material for evaluating teaching materials and methods. The suitability of the material delivered through a suitable method, students will easily understand and can prevent the emergence of student difficulties in mastering the material. There are many ways to overcome geometry proof errors, namely understanding the context of the problem, using logical arguments, frequent practice problems, discussions with friends or lecturers to get feedback from the results of the proof that has been done. According to Llinares, that students or prospective teachers need to be aware of the connections between specific geometrical facts when they construct a proof by linking visualisation to formal reasoning (Llinares & Clemente, 2019).

CONCLUSION

Based on the results and discussion that has been described, it is concluded that students' errors in working on geometry proof problems that many do is the error of transforming the problem. The details of errors and their characteristics are errors in reading mathematical symbol representations with an average of 15.81%, with the characteristics of errors that students do not write geometry symbols according to the rules. Comprehension error of proof problem with an average of 4.78%, with the characteristics of the error that students do not write in full the information obtained, what will be proven and the construction of images in the problem. Transformation error of proof problem with an average of 26.84%, with error characteristics that students fail to determine and choose the right proof procedure and are incomplete in applying the chosen plan. Process skill errors of proof problem with an average of 29.04%, with error characteristics that students are wrong in determining the reasons for each statement. Encoding or answer writing errors with an average of 23.53%, with the characteristics of the error that students are not correct in concluding the final result of the proof. As for the solution to prevent repeated errors, students should be reminded to understand the problem of proof carefully and students should have the initiative to often practice geometry proof problems.

The researcher realizes that there are shortcomings and obstacles in the research process in the form of a less in-depth interview process and the limited knowledge and experience of researchers in this scientific paper, especially in analyzing each item. Therefore, for other researchers who want to conduct similar research in order to better prepare interview guideline instruments well and study more deeply in order to get broader information. The results of this study also need follow-up to produce new research. The results of this study should be used as evaluation material for related parties so that the learning outcomes of geometry courses are well implemented.
Declarations

Author Contribution: SANA: Conceptualization, Writing - Initial Draft, Formal analysis, Methodology, Editing, and Visualization; IM: Writing - Review & Editing, Validation and Supervision.

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