Exploration of conceptual understanding and values in mathematics among prospective mathematics teachers

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Abstract: The study examines the relationship between understanding mathematical concepts and mathematical values among prospective teachers at a state university in Central Java. The research employed a qualitative approach with content analysis, involving a mathematical concept understanding test and in-depth interviews with students at high, medium, and low ability levels. The results show that students with high ability tend to possess strong mathematical values like accuracy, perseverance, and curiosity, which enable them to grasp complex mathematical concepts. In contrast, students with low ability struggle to comprehend basic mathematical concepts and often rely on rote calculation and formula use. The findings suggest that differentiated learning strategies are necessary to cater to the diverse levels of mathematical concept understanding among prospective teachers. Enhancing mathematical values can be a key factor in improving conceptual understanding in mathematics education.

Keywords: Conceptual understanding, Prospective teacher, Mathematical values

INTRODUCTION

Mathematics plays a crucial role in the advancement of science and technology. A strong understanding of mathematical concepts forms an essential foundation for developing critical thinking skills and problem-solving abilities, which are vital in various aspects of life (Novikasari, 2022). Therefore, mathematics education in schools is a cornerstone for establishing a robust mathematical understanding among students. Without a strong grasp of mathematical concepts, students may struggle to apply mathematics in daily life and related academic fields.

Prospective mathematics teachers hold a strategic role in the teaching and learning process of mathematics. They are expected to possess a deep understanding of mathematical concepts and instill positive values about mathematics in their students (Dede et al., 2024). These mathematical values encompass perceptions of the importance of mathematics, attitudes towards learning mathematics, and beliefs about one's ability to master mathematics. These values are crucial in shaping students' attitudes and motivations towards mathematics, ultimately influencing their learning outcomes.

The mathematical beliefs prospective teachers hold are crucial in shaping their instructional approaches. Teachers who maintain a growth mindset and understand the significance of mathematics tend to be more engaged and effective in their teaching practices (Bostwick, et al., 2020). Conversely, negative views or lack of confidence among prospective teachers regarding their teaching abilities in mathematics can adversely affect classroom learning.
Therefore, understanding the mathematical values of prospective teachers is essential to ensure high-quality teaching (Novikasari et al., 2024). According to Kalogeropoulos et al. (2021), in developing values in mathematics education, prospective teachers must understand them and then choose appropriate strategies to impart them to students through mathematics instruction.

This study explores the mathematical values held by prospective teachers and how these values relate to their understanding of mathematical concepts. Understanding the relationship between mathematical values and conceptual understanding aims to provide comprehensive insights into factors influencing the effectiveness of prospective teachers in teaching mathematics. Furthermore, this research can help identify areas for improvement in teacher education programs, thereby supporting prospective teachers in developing better attitudes and understanding towards mathematics.

Moreover, this study is expected to significantly contribute to the field of mathematics education, particularly in developing educational programs and training for prospective teachers. By identifying and reinforcing positive values about mathematics and enhancing conceptual understanding, prospective teachers can be better prepared and more confident in their teaching roles. This process, in turn, can improve the quality of mathematics education in schools and empower students to become more competent and confident in mastering mathematics.

Mathematical values refer to individuals' beliefs, attitudes, and perceptions towards mathematics. These values encompass various aspects, such as the perception of the importance of mathematics in everyday life, attitudes towards learning mathematics, and beliefs about one's ability to understand and master mathematics. Seah et al. (2021) note that the selection of values in mathematics education evolves and grows through influence from various sources. Ernest (1991) categorizes mathematical values into three main levels: instrumental, intrinsic, and social. Instrumental values refer to the view that mathematics is a useful tool for solving practical problems in daily life and across various scientific fields. Intrinsic values relate to an appreciation for the beauty and orderliness of mathematics itself, where individuals enjoy the intellectual challenges presented by mathematics. Meanwhile, social values reflect the belief that mathematics is crucial to societal progress and technological advancement. These values are crucial in shaping individuals' attitudes and motivations towards mathematics, influencing their learning and teaching it (Hannula, 2006).

Conceptual understanding in mathematics refers to comprehending, interpreting, and applying mathematical concepts correctly and effectively. Kilpatrick et al. (2001) state that conceptual understanding involves several key components: relational understanding, procedural understanding, and conceptual understanding. Relational understanding is perceiving and understanding relationships among different mathematical concepts. Procedural understanding involves the ability to apply mathematical procedures and algorithms correctly. Meanwhile, conceptual understanding entails understanding the meaning and foundations of mathematical concepts rather than merely memorizing formulas and procedures. A strong conceptual understanding enables individuals to solve mathematical problems more flexibly and creatively and to transfer mathematical knowledge to different contexts.

According to Seah and Wong (2012), individuals' mathematical values can influence their ability to understand mathematical concepts. Based on the motivation and learning theory articulated by Dweck (1986), individuals' beliefs and attitudes towards mathematics can affect their efforts and persistence in learning mathematics. Individuals with positive values towards mathematics tend to be more motivated to deeply understand mathematical concepts. Research by Pajares and Graham (1999) indicates that individuals' beliefs in their mathematics self-efficacy can influence how they learn and understand mathematics. Individuals with high self-efficacy tend to be more confident in facing mathematical challenges and more persistent in seeking correct conceptual understanding.
Social values also play a significant role in shaping conceptual understanding in mathematics. Individuals who believe mathematics is crucial for societal progress may be more motivated to understand mathematical concepts well because they see greater value in learning mathematics (Vos et al., 2024). These social values can also encourage individuals to participate in mathematics learning communities, such as study groups or class discussions, which can enrich their understanding of mathematical concepts.

Understanding the relationship between mathematical values and conceptual understanding has important educational implications. Approaches to learning that focus on developing and nurturing positive values towards mathematics can enhance students' motivation and learning outcomes. Teachers play a key role in shaping these values through their methods of teaching mathematics and interactions with students. Furthermore, education programs and training for prospective mathematics teachers should include the development of positive mathematical values. Prospective teachers with positive values towards mathematics are likely to be more effective in teaching mathematics and capable of instilling positive attitudes towards mathematics in their students. These training programs should also emphasize the importance of a deep conceptual understanding of mathematics, enabling prospective teachers to teach confidently and competently.

The theory of this study demonstrates that mathematical values and conceptual understanding are interrelated and play crucial roles in the learning process. By understanding and developing both aspects, we can enhance the quality of mathematics education and prepare a more competent and confident generation to master mathematics.

METHOD

This study employs a qualitative research method with content analysis. This approach was chosen to identify and analyze prospective teachers' mathematical values about their understanding of mathematical concepts. Content analysis allows researchers to delve deeply into candidates' perspectives on mathematics through data generated from tests and interviews (Krippendorff, 2018).

Participants

The subjects of this study are prospective mathematics teacher students currently enrolled at a state university in Central Java, totaling 12 students. This research involves students in the final stages of their studies, expected to possess a sufficient understanding of mathematical concepts and teaching experience. In this qualitative study, the selection of subjects was carefully conducted to ensure representative variation in both mathematical concepts understanding, and the mathematical values held. Participants were chosen from a state university in Central Java to reflect a diverse range of educational backgrounds and experiences within this geographical region. To ensure this variation, we employed specific criteria for inclusion and exclusion. These criteria included academic performance in mathematics-related courses, prior teaching experience, and demonstrated interest in mathematics education. By selecting participants who met these criteria, we aimed to capture a comprehensive and representative understanding of the different levels of mathematical concept comprehension and the spectrum of mathematical values among prospective mathematics teachers.

Initially, two prospective mathematics teacher students were selected for each category of mathematical concept understanding test results. This selection aimed to identify the mathematical values held by each category. If two students within one category exhibited
significantly different characteristics, additional subjects from that category were included until similar characteristics were identified. This approach ensures that the collected data reflects consistent and reliable patterns, yielding more accurate insights into the relationship between mathematical values and conceptual understanding among prospective teachers.

**Research Instruments**

A mathematical concept understanding test was utilized to measure students' abilities in understanding mathematical concepts, with indicators including defining concepts, identifying and providing examples or non-examples of concepts, developing mathematical connections between various ideas, comprehending how mathematical ideas interrelate to build comprehensive understanding, and applying mathematics in contexts beyond mathematics (Novikasari, 2022). This test was designed to encompass various relevant mathematical concepts and was expected to classify students into three levels: high, medium, and low, based on their test results. Based on the conceptual understanding of test results, the researcher initially selected two prospective mathematics teacher students from each level (high, medium, low) for interviews. This selection considered variations in mathematical values students might possess at each level. The researcher then added research subjects until similar characteristics were obtained for each category.

In-depth interviews were conducted to explore students' views, attitudes, and beliefs about mathematics. These interviews focused on the mathematical values they adhere to, such as perceptions of the importance of mathematics, attitudes towards learning mathematics, and beliefs about their abilities to understand and teach mathematics. Interviews were conducted in stages until saturation was reached, where no new information emerged from additional interviews.

**Data Collection and Analysis**

Data obtained from conceptual understanding tests and in-depth interviews were analyzed using content analysis techniques. The analysis process involved several stages. First, interviews were recorded and transcribed verbatim to ensure data accuracy. Second, transcript data were analyzed by coding themes related to mathematical values and conceptual understanding abilities. Third, identified codes were grouped into broader categories to identify patterns and relationships between mathematical values and conceptual understanding. Finally, the categorized results were analyzed in-depth to understand the characteristics of mathematical values held by prospective teacher students at each level of conceptual understanding (Krippendorff, 2018). The researcher used data triangulation techniques to ensure data validity and reliability, comparing results from conceptual understanding tests with in-depth interviews. Additionally, member checking was conducted with participants to confirm findings derived from interviews.

**RESULTS AND DISCUSSION**

Results

This research identifies prospective mathematics teachers' mathematics values and their ability to understand mathematical concepts. The research involved a detailed analysis of the participants' mathematical values such as accuracy, perseverance, and curiosity, and correlated these values with their conceptual understanding of mathematics. Table 1 is a description of the research results based on prospective teachers' level of ability and views about mathematics. Table 1 provides a comprehensive breakdown of these findings, highlighting the distinct differences in mathematical values and conceptual understanding among prospective teachers across different ability levels. These results underscore the importance of fostering strong mathematical values to enhance conceptual understanding and suggest the need for
differentiated learning strategies to address the diverse needs of prospective mathematics teachers.

Table 1. Level of Understanding of Mathematical Concepts and Mathematical Values

<table>
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<tr>
<th>Understanding Math Concept</th>
<th>Subject</th>
<th>Math Values</th>
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<tbody>
<tr>
<td>High</td>
<td>APR, DSA, SZI, DSM</td>
<td>Fun, thorough, challenging, discovering new things, practice, exact knowledge, understanding concepts</td>
</tr>
<tr>
<td>Medium</td>
<td>NCK, SWN, ENF</td>
<td>Complicated science, difficult, requires perseverance, moody, a little fun</td>
</tr>
<tr>
<td>Low</td>
<td>EBO, RAH, MNZ, FNK, SNH</td>
<td>Counting, difficult because it requires precision, confusing, uses formulas</td>
</tr>
</tbody>
</table>

Student teachers at high proficiency levels, such as APR, DSA, SZI, and DSM, demonstrate deep mastery of complex mathematical concepts. They approach mathematical material with enthusiasm and see it as a fun challenge. For them, mathematics is not just a set of rules or formulas but an opportunity to discover new things and develop deep analytical thinking skills. They use a systematic and careful approach to solving mathematical problems, combining intensive practice with a strong understanding of concepts. Their views on mathematics reflect a sense of attraction to the scientific certainty it offers, where accuracy and thoroughness are the main values, they believe in. During the interview with DSA regarding understanding quadratic functions, DSA's responses reflected a deep engagement with the concept and an appreciation for its real-world applications. DSA highlighted that "The concept of quadratic functions can be found in real-world problems involving the use of graphs, numbers, or symbols in mathematical modeling." This statement reveals that DSA possesses a mathematical value centered on discovering new things. DSA's approach to quadratic functions underscores the importance of exploration and the practical application of mathematical concepts, demonstrating an intrinsic motivation to uncover and understand how to solve real-life problems. This perspective aligns with a broader educational goal of fostering curiosity and a continuous quest for knowledge among prospective mathematics teachers.

Students at medium ability levels, exemplified by NCK, SWN, and ENF, demonstrate a more ambivalent attitude towards mathematics. They recognize the complexity and level of difficulty associated with deeper mathematical concepts. For them, studying mathematics requires a lot of perseverance and dedication, often balanced by feeling moody or inconsistent with the material they are exposed to. Despite challenging some aspects of mathematics, their views tend to recognize the need to engage intensively in the learning process. They can demonstrate sufficient ability to understand basic concepts, although they often need additional help to overcome obstacles. During the interview with SWN regarding the understanding of quadratic functions, SWN expressed that mathematics can sometimes be stressful, challenging, and difficult, especially when faced with complex problem-solving tasks. However, SWN also implied that these difficulties can be overcome with perseverance and a good learning approach, making mathematics less daunting. SWN's perspective highlights the importance of resilience and effective learning strategies in mastering mathematical concepts, suggesting that initial challenges may be significant with dedication and the right mindset. This viewpoint reflects the value of persistence in learning and the belief that overcoming obstacles in mathematics is possible through sustained effort and proper guidance.

Student teachers at lower proficiency levels, such as EBO, RAH, MNZ, FNK, and SNH, face mathematics with greater challenges. They often have difficulty understanding even basic mathematical concepts. For them, mathematics is usually seen as a skill that requires high precision in calculating and applying formulas. Their view of mathematics tends to be negative, and they feel trapped in solving confusing and difficult-to-understand problems. They may feel less confident in their mathematical abilities, which can hinder their future progress in understanding and teaching this subject. During the interview with RAH regarding
understanding quadratic functions, RAH emphasized that the key to mastering mathematics lies in a deep understanding of formulas, which students can achieve through repeated practice. However, RAH also expressed difficulty in relating mathematical concepts to real-life contexts. This perspective underscores the importance of continuous training and repetition in gaining proficiency in mathematics. Yet, it also highlights a common challenge many students face: the ability to see the practical applications of mathematical concepts in everyday life. RAH’s insights suggest a need for instructional approaches that focus on repetitive practice and contextualize mathematics in real-world scenarios to enhance conceptual understanding and relevance.

Discussion

The findings highlight significant diversity in the perspectives and competencies of student teachers about mathematics. Students at high proficiency levels exhibit robust capabilities in comprehending intricate mathematical concepts and maintain a positive attitude towards the subject. Conversely, students with moderate abilities demonstrate commendable efforts in overcoming mathematical challenges yet frequently harbor mixed feelings towards their study content. Meanwhile, students at lower proficiency levels require targeted interventions to surmount hurdles in grasping fundamental mathematical concepts and to cultivate a more positive outlook on the subject. These observations underscore the necessity of employing differentiated teaching approaches in mathematics education, tailored to accommodate varying proficiency levels and to facilitate optimal development in understanding and teaching mathematics. This study offers a comprehensive synthesis of findings regarding the diversity in perspectives and competencies among student teachers across different proficiency levels in mathematics. The research highlights significant variations in conceptual understanding and mathematical values among participants categorized into high, medium, and low proficiency groups. High-proficiency individuals demonstrated a robust ability to define concepts, make connections across mathematical ideas, and apply mathematics in practical contexts. In contrast, those in the low proficiency group showed challenges in these areas, often needing help to relate mathematical concepts to real-world scenarios. These findings underscore the pivotal role of mathematical values in shaping attitudes and abilities in mathematics education. They emphasize the importance of fostering a solid conceptual foundation and cultivating an appreciation for the practical applications of mathematics among future educators. This synthesis contributes valuable insights into tailoring educational strategies that effectively support diverse learning needs and promote a deeper engagement with mathematics among prospective teachers.

The study identified significant disparities in mathematics scores and the ability to comprehend mathematical concepts among student teachers, stratified by their proficiency levels. These findings corroborate prior research emphasizing the influence of mathematical values on students' perceptions and competencies in this field (Hwang & Son, 2021). According to Hwang and Son (2021), students with higher mathematics scores exhibit more excellent proficiency in grasping complex mathematical concepts, while those with lower scores encounter obstacles in comprehending fundamental mathematical principles. Furthermore, the study aligns with the conclusions drawn by Moussa and Saali (2022), who assert that positive attitudes toward mathematics and values such as thoroughness and diligence contribute significantly to enhanced mathematical learning abilities.

The findings of this study hold significant educational implications for mathematics teacher education programs. The research underscores the importance of differentiated teaching approaches by highlighting the diverse perspectives and competencies among student teachers across varying proficiency levels. Tailoring instructional strategies to accommodate the distinct learning needs of high, medium, and low proficiency groups can effectively support their development in mathematics education (Insorio, 2024). For high-proficiency individuals, emphasizing advanced problem-solving techniques and encouraging the application of
mathematical concepts in real-world scenarios can further enhance their skills and confidence. Medium proficiency learners may benefit from targeted interventions that strengthen foundational knowledge and foster connections between mathematical ideas. Meanwhile, addressing low-proficiency students' challenges through remedial support and hands-on learning experiences can help bridge gaps in understanding and cultivate a positive attitude towards mathematics. By implementing these differentiated approaches, mathematics teacher education programs can better prepare future educators to meet the diverse learning needs of students and promote a deeper engagement with mathematics in educational settings. By fostering positive attitudes and reinforcing foundational mathematical values, educators can potentially enhance students' motivation and proficiency in mathematics. Furthermore, the research underscores the critical role of educators in nurturing a conducive learning environment that fosters positive attitudes towards mathematics across all proficiency levels. Educators can empower student teachers to develop a comprehensive understanding and proficiency in mathematics by promoting a supportive and inclusive approach to teaching mathematics—this is important in recognizing and addressing the varying levels of mathematics understanding concepts and values towards mathematics among student teachers. Through targeted educational strategies and supportive teaching practices, educators can cultivate a positive learning experience that enhances students' confidence and competence in mathematics. However, these findings also show differences from several other studies, which emphasize social and environmental factors in mathematics learning more. For example, recent research on the impact of a growth mindset on mathematics achievement has shown that intrinsic motivation, self-efficacy, and reduced mathematics anxiety are crucial for improving mathematics skills (Dong et al., 2023). These studies underscore the complexity of factors that influence understanding mathematical concepts, including how individual values interact with social context and learning motivation (Dong et al., 2023; Megan & Baker, 2023).

The implications of this research can benefit the future development of mathematics teacher education programs. By understanding that the mathematics values held by prospective teachers play a role in their ability to understand and teach this subject, teaching approaches can be adapted to support all levels of ability. The mathematics teacher education curriculum must emphasize lengthening values such as thoroughness, caution, and enthusiasm for discovering new things. Emphasis should be placed on learning strategies that can assist students with lower abilities overcome challenges to comprehend fundamental mathematical concepts.

The findings presented in this study suggest that future research could explore several promising avenues to build upon these insights. Longitudinal studies could investigate the sustained impact of educational interventions on the mathematical values and proficiency levels of student teachers over time. By tracking participants' progress and experiences beyond initial interventions, researchers could gain deeper insights into the long-term effectiveness of different teaching approaches in shaping attitudes towards mathematics. Furthermore, there is a need for more research to delve into the complex factors influencing mathematical learning among prospective educators. This research explores socio-cultural influences, pedagogical strategies, and individual motivational factors contributing to varying mathematical proficiency and engagement levels. Investigating how teacher educators can leverage prospective teachers using technological advancements and innovative teaching methods to enhance mathematical understanding and application in diverse educational contexts would provide valuable insights for improving mathematics teacher education programs. These future research directions aim to contribute to a more comprehensive understanding of the multifaceted nature of mathematical learning and inform targeted interventions that support effective teaching and learning practices in mathematics education.
CONCLUSION

This research concludes that the mathematical values held by student teachers are related to their ability to understand mathematical concepts. Students with high abilities tend to have a positive view of mathematics, driven by values such as rigor and a passion for discovering new things. In contrast, students with lower abilities face challenges in understanding mathematical concepts, with a more negative view of this subject. These findings indicate the need for a differential learning approach in mathematics teacher education, which not only strengthens mathematical values but also builds deep conceptual understanding among student teachers with varying levels of ability. Future research could expand the scope to validate these findings in other educational contexts.

Based on the finding that different mathematics values are found in the ability to understand different mathematical concepts among student teachers, future research can expand the scope of this study to various other educational contexts. Future research could consider longitudinal studies to observe how mathematics values and understanding of mathematics concepts develop over time and what educational interventions are most effective in shaping positive values and deep understanding among prospective mathematics teachers. Research could also explore the impact of differential learning approaches in strengthening mathematics values and improving conceptual understanding in students of varying ability levels. Thus, further research will validate current findings and provide practical guidance for developing more effective and inclusive teacher education programs.

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REFERENCES


